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Open and Closed Universes in Brane-World Cosmology

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Abstract

We find two types [called (S) and (A)] of new vacuum solutions of open, flat, and closed universes which are inflating in the brane-world scenario. We show that the warp factor of the stabilized metric is universal for the three different kinds of universes. For (S) type solution, we show that one positive-tension brane universe solution is admitted as well as two positive tension brane solution even if we consider the vacuum solution. For (A) type solution, we find that the inflating bulk solutions have black hole like regions and that the full extended space is the R-S solution.

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1 Introduction

There has been considerable interest in the possibility of large extra-dimension scenario and its applications to cosmology after it was found that this scenario can resolve the long-lasting problem of the hierarchy: The huge gap between the electroweak scale M_W and the Planck scale M_P, $M_P/M_W \sim 10^{16}$. By allowing the extra dimensions to be *significantly larger* than the Planck scale, one can try to relate the electroweak scale to the fundamental higher-dimensional Planck scale, if the matters are confined to our 4-dimensional space-time [1, 2, 3].

Randall and Sundrum [R-S] [4] suggested an alternative solution to the hierarchy problem where the metric has exponential dependence on the extra-dimension y:

$$ds^2 = e^{-2\lambda r_c|y|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 dy^2, \tag{1}$$

where y ranges $-1 \leq y \leq 1$ and mirror symmetry for $y = 0$ is assumed. This metric satisfies the vacuum Einstein equation with the following relations between the cosmological constants

$$\sqrt{\frac{-\Lambda}{24M^3}} = \frac{\Lambda_{hid}}{24M^3} = -\frac{\Lambda_{vis}}{24M^3}, \tag{2}$$

where Λ, Λ_{hid}, Λ_{vis} are the cosmological constants of the bulk, the hidden brane, and the visible brane, respectively. In this model, the 4-dimensional Planck scale is given by

$$M_P^2 = \frac{M^3}{\lambda}[1 - e^{-2\lambda r_c}] \tag{3}$$

and, for large λr_c, it is λ rather than r_c that determines M_P. R-S argued that due to the warp factor $e^{-\lambda r_c|y|}$ which has different values at the hidden brane $y = 0$ and at the visible brane $y = 1$, any mass m_0 on the visible brane corresponds to a physical mass $m = m_0 e^{-\lambda r_c}$ and the moderate value $\lambda r_c \sim 37$ can produce the huge hierarchy $M_P/M_W \sim 10^{16}$. Thus the gauge hierarchy problem was converted to the problem of fixing the size of the extra dimension. Moreover, this warp factor opened the possibility of infinitely large extra-dimensions [5] due to the localization of the gravity in addition to that of the matter. This scenario is supported by the heterotic M-theory, whose field theory limit is the 11-dimensional supergravity compactified on S_1/Z_2 with supersymmetric Yang-Mills fields living on two boundaries [6, 7, 8, 9].

On the other hand, R-S solution has two well-known defects: one is that it contains a negative tension brane and the other is that it needs a fine tuning (3) of the cosmological constants of the bulk and of the branes.

For the first problem, it is known that this can be overcome by permitting different cosmological constants [10, 11] for three spatial regions or by considering more complicated theories with matter fields [12, 13]. But it seems that these approaches are too artificial in our viewpoint: This is one of our motivations.

In regard to the second problem, several authors [14, 15, 16] have considered theories without fine-tuning (2) and they have found the inflationary solutions. However, it was noticed [17] that the Hubble parameter H of these inflationary solutions in the brane-world cosmology has a different behavior from that of the usual 4-dimensional Friedmann equation; in particular, the Hubble parameter is proportional to the energy density on the brane instead of the familiar dependence $H \sim \sqrt{\rho}$. Several authors suggested that this problem can be cured [14, 15, 16, 18, 19] by requiring the cancellation of the leading brane tension squared term Λ^2_{brane} and the bulk cosmological constant Λ with matters on the brane. On the other hand, Kanti, Kogan, Olive, and Pospelov [12] derived sufficient conditions which ensure a smooth transition to conventional cosmology and Newton's law on the brane. The condition is the existence of $T_{55} \propto (-\rho + 3p)/L + O(\rho^2)$, whose value is responsible for the stabilization of the dilaton. We will implicitly assume, in this paper, these types of resolution for the inflating solution and we will not discuss it further. We shall assume that there exists some stabilization mechanism, and do not give a specific prescription.

Rather in this paper, we are concerned about the generalizations of the (vacuum) inflating solutions to the spatially open and the spatially closed universes, in contrast to the previous works which have been restricted to the spatially flat universes. One of the notable features of our new solutions is the existence of a compact vacuum solution with one positive tension brane, and the existence of the horizon on the bulk of the inflationary solutions which has not been known yet.

In Sec. 2, we try to make a general effective action which describes the brane cosmological systems. We use the $4+1$ splitting and the conformal transformation. We then choose a gauge condition which should be satisfied by vacuum solutions. In Sec. 3 we generate several solutions which are inflationary. We find two types of inflating solutions of the open, flat, and closed universes with different warp factors: $\cosh^2(N\gamma|y| + \beta)$ and $\sinh^2(N\gamma|y| + \beta)$; we find that $\cosh^2$ type solution allows one or two positive tension branes as well as the one positive and one negative tension branes depending on the initial data. We also give the coordinate transformations between the R-S solution and our $\sinh^2$ type inflationary solution. It is found that there exist an event horizon, which can be a past horizon or a future horizon, but not both. Finally, we summarize our results and present some comments in Sec. 4.

2 Setting

We consider the vacuum solution of the brane world in D=5. The fifth dimension $x^4 = y$ satisfies reflection symmetry at $y = 0$. The five dimensional indices are denoted by $M, N = 0, 1, \cdots, 4$ and the four-dimensional indices of space-time by $a, b = 0, 1, 2, 3$. The range of y is restricted to $-1 \le y \le 1$.

The 5-dimensional metric can be written, in general, as

$$ds^2 = G_{AB}dx^A dx^B = e^{-2\sigma} g_{ab}(dx^a + N^a dy)(dx^b + N^b dy) + N^2 dy^2, \tag{4}$$

with $\sqrt{G} = Ne^{-4\sigma}\sqrt{g}$. The unit normal to the time-like section of $y =$ constant is denoted by $n_A = (0,0,0,0,N)$ and $n^A = G^{AB}n_B = (-N^a/N, 1/N)$. The induced metric on this 4-dimensional section is given by

$$h_{AB} = G_{AB} - n_A n_B. \tag{5}$$

From the Gauss-Codazzi relation, 5-dimensional curvature scalar ${}^{(5)}R$ can be expressed by its 4-dimensional curvature ${}^{(4)}R$ and the extrinsic curvature $\bar{K}_{AB}$ of the section up to total derivatives:

$${}^{(5)}R ={}^{(4)} \bar{R} + \bar{K}^2 - \bar{K}_{AB}\bar{K}^{AB} + \text{total derivatives}, \tag{6}$$

where $\bar{K}_{AB} = \frac{1}{2}\mathcal{L}_n G_{AB}$ is the Lie derivative of the metric.

Using the conformal transformation $h_{ab} = e^{-2\sigma}g_{ab}$, one obtains ${}^{(4)}\bar{R}$:

$${}^{(4)}\bar{R} = e^{2\sigma}\left\{R + 6g^{ab}\nabla_a\nabla_b\sigma - 6\nabla_a\sigma\nabla^a\sigma\right\}, \tag{7}$$

where R is the intrinsic curvature of g_{ab}. The extrinsic curvature is given by

$$\bar{K}_{ab} = \frac{1}{2}\mathcal{L}_n h_{ab} = \frac{1}{2}\mathcal{L}_n(e^{-2\sigma}g_{ab}) = -(n^C\partial_C\sigma)e^{-2\sigma}g_{ab} + e^{-2\sigma}K_{ab}, \tag{8}$$

where K_{ab} is the extrinsic curvature of $y =$ constant section with metric g_{ab}.

Thus the Einstein action for 5-dimensional gravity including the cosmological constant term can be written as

$$\begin{aligned} S &= \frac{1}{16\pi}\int_M d^5x\,\sqrt{-G}\left[M^3R - \frac{1}{2}\Lambda\right] + \sum_{\text{branes}} S_i \\ &= \frac{M^3}{16\pi}\int d^5x\sqrt{g}e^{-2\sigma}\Big\{N[R + 6\nabla_a\sigma\nabla^a\sigma] - 6g^{ab}\nabla_a N\nabla_b\sigma \\ &+ Ne^{-2\sigma}\Big[12(n^C\partial_C\sigma)^2 - 6K(n^C\partial_C\sigma) + K^2 - K^{ab}K_{ab}\Big] - \frac{\Lambda}{2M^3}Ne^{-2\sigma}\Big\} + S_B, \end{aligned} \tag{9}$$

where $S_{\rm i}$, Λ, and M are the action for the brane, cosmological constant of the bulk, and the fundamental gravitational scale of the model, respectively. S_B is the sum of all the boundary terms in the canonical form of the action. The brane action S_i is given by

$$S_i = -\frac{1}{32\pi}\int d^4x\; e^{-4\sigma}\sqrt{-g_i}\;\Lambda_i + S_{\text{matter}}, \tag{10}$$

and the variation of S_i by,

$$16\pi\delta S_i = \int d^5x\; e^{-4\sigma}\sqrt{-g}\Lambda_i\delta(y - y_i)\left[2\delta\sigma + \frac{1}{4}g_{ab}\delta g^{ab}\right] + 16\pi\delta S_{\text{matter}}. \tag{11}$$

The extrinsic curvature K_{ab} is explicitly written as

$$K_{ab} = \frac{1}{2}\mathcal{L}_n g_{ab} = \frac{1}{2N}\left[\partial_y g_{ab} - \nabla_a N_b - \nabla_b N_a\right]. \tag{12}$$

We now consider a gauge choice that simplifies the action. We choose the metric g'_{ab} to satisfy

$$\partial_y g'_{ab} = \partial_y g_{ab} - \nabla_a N_b - \nabla_b N_a. \tag{13}$$

This choice of gauge can be justified because the gauge degree of freedom $x^a \longrightarrow x^a + \xi^a$ leads to

$$g'_{ab} = g_{ab} + \xi_{a;b} + \xi_{b;a}. \tag{14}$$

In this gauge (13), the shift function N^a should satisfy,

$$\nabla_a N_b + \nabla_b N_a = 0 \tag{15}$$

which means that N_a is a Killing vector of g_{ab} or zero. If N_a is a Killing vector of g_{ab}, one can set N_a proportional to a coordinate vector e_a. Then simple reparametrization of x^a coordinates can remove N_a. This choice, $N_a = 0$, can be justified because the spatial coordinates can usually be adjusted to be orthogonal to y, but N_t can not be gauged away when there is energy exchange in y direction. If one is interested in the vacuum solutions as in this paper, N_t can be trivially gauged away. We thus set $N_a = 0$ from now on.

With this gauge choice the action becomes

$$\begin{aligned} S &= \frac{M^3}{16\pi}\int d^5x\sqrt{g}e^{-2\sigma}\Big\{N[R + 6\nabla_a\sigma\nabla^a\sigma] - 6g^{ab}\nabla_a N\nabla_b\sigma \\ &+ \frac{1}{N}e^{-2\sigma}\left[12(\partial_y\sigma)^2 - 3g^{ab}\partial_y g_{ab}(\partial_y\sigma) + \frac{1}{4}[(g^{ab}\partial_y g_{ab})^2 - g^{ac}g^{bd}\partial_y g_{ab}\partial_y g_{cd}]\right] \\ &- \frac{\Lambda}{2M^3}Ne^{-2\sigma}\Big\} + S_B. \end{aligned} \tag{16}$$

Note that there is a time-derivative term of N in this action in contrast to the usual time-slicing formulation. The variation of this reduced action reproduces the Einstein equations, except for the fact that the part of the Einstein equation, $G_{\mu 5} = 0$, which corresponds to the variation of N_a.

3 Robertson-Walker models

Since we are interested in the cosmological solutions, we assume that the three-dimensional spatial section is homogeneous and isotropic. The most general metric of this kind is given by

$$g_{ab}dx^a dx^b = -d\tau^2 + a^2(\tau, y)d\xi^2_{(3)}, \tag{17}$$

where $d\xi_{(3)} = \chi_{ij}dx^i dx^j$ is completely symmetric spatial geometry. The intrinsic and the extrinsic curvature of the metric are

$$R = 6\left[\frac{\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2\right] + \frac{6k}{a^2}, \qquad K^a_b = \frac{1}{N}\mathrm{diag}\left(0, \frac{a'}{a}, \frac{a'}{a}, \frac{a'}{a}\right), \tag{18}$$

where $k = 1, 0, -1$ for closed, flat, open spatial geometries, respectively. The overdot ($\dot{a}$) denotes the time derivative and the prime (a') the derivative with respect to y.

We measure the scale of the universe by using the fundamental scale of the system, $a(t, y) = e^r/M$, and then perform change of variable $R = r - \sigma$. Then the effective action reads

$$\begin{aligned} S &= \frac{6}{16\pi}\int \sqrt{g} d^5x N e^{3R-\sigma}\left\{-e^{2\sigma}\left[\dot{R}^2 + \frac{\dot{N}}{N}\dot{R}\right]\right. \\ &+ \left.\frac{1}{N^2}\left(R'^2 - \sigma' R'\right) - \frac{\Lambda}{12M^3} + kM^2 e^{-2R}\right\} + S_B. \end{aligned} \tag{19}$$

The equations of motions are given by

$$\ddot{R} + 2\dot{R}^2 + \dot{\sigma}\dot{R} - \frac{e^{-2\sigma}}{N^2}(R'^2 - R'\sigma') + \left[-\frac{\Lambda}{12M^3} + kM^2 e^{-2R}\right] e^{-2\sigma} = 0, \tag{20}$$

$$\begin{aligned} &\dot{R}^2 + \dot{R}\frac{\dot{N}}{N} + \frac{1}{N^2}e^{-2\sigma}\left[-2R'^2 - \frac{N'}{N}R' - R''\right] + \left(-\frac{\Lambda}{12M^3} + kM^2 e^{-2R}\right) e^{-2\sigma} \\ &\quad = \frac{e^{-2\sigma}}{N}\sum_i \frac{\Lambda_i}{12M^3}\delta(y - y_i), \end{aligned} \tag{21}$$

$$\begin{aligned} &2\ddot{R} + 3\dot{R}^2 + 2\dot{R}\dot{\sigma} + 2\dot{R}\frac{\dot{N}}{N} + \dot{\sigma}\frac{\dot{N}}{N} + \frac{\ddot{N}}{N} + \frac{1}{N^2}e^{-2\sigma}\left[-2R'' + \sigma''\right. \\ &\left.\frac{N'}{N}(2R' - \sigma') - 3R'^2 + 2R'\sigma' - \sigma'^2 - \frac{\Lambda N^2}{4M^3}\right] + kM^2 e^{-2R}e^{-2\sigma} \\ &\quad = \frac{e^{-2\sigma}}{N}\sum_i \frac{\Lambda_i}{4M^3}\delta(y - y_i), \end{aligned} \tag{22}$$

where we have used the fact that

$$\delta S_i = -\frac{6}{16\pi}\int d^5x \frac{\Lambda_i}{12M^3} e^{3R-\sigma}\delta(y - y_i)\left[3\delta R - \delta\sigma\right]. \tag{23}$$

If there exists a stabilization procedure [13], N will become independent of t, and the y dependence can be removed by coordinate transformation. This leads to a constraint equation which is independent of time derivatives if one sets $N =$ constant. This is the same kind of equation as the Hamiltonian constraint, which implies the non-dynamical nature of the lapse function. We then obtain

$$R'' + R'^2 - \frac{1}{3}\sigma'' + \frac{1}{3}\sigma'^2 + \frac{\Lambda N^2}{18M^3} = -\frac{N}{9}\frac{\Lambda_i}{M^3}\delta(y - y_i) \tag{24}$$

from Eqs. (20), (21), and (22). Eq. (24) implies an interesting point: Since the equation is k independent, the y-dependence of the open, flat, and closed universe are the same if the 5-th direction is stabilized.

We now assume $\sigma' = -R'$, which corresponds to $a' = 0$, and solve Eq. (24) at the bulk ($-1 < y < 1$). This leads to the two sets of solutions:

$$\begin{aligned} \sigma_{S,A}(y,t) &= -\ln[e^{N\lambda(y-y_0(t))} \pm e^{-N\lambda(y-y_0(t))}] + \sigma_0(t), \\ R_{S,A}(y,t) &= \ln[e^{N\lambda(y-y_0(t))} \pm e^{-N\lambda(y-y_0(t))}] + R_0(t), \end{aligned} \tag{25}$$

where the subscripts S and A refer to $+$, $-$ sign, respectively, and $\lambda^2 = \frac{-\Lambda}{24M^3}$. Since the dependence on $\sigma_0(t)$ can always be absorbed by redefining the time coordinate, we set $\sigma_0(t) = 0$.

By substituting both sets of Eqs. (25) to the first and second equations of (20) one obtains the following sets of equations:

$$\dot{y_0}(t) = 0, \tag{26}$$

$$\dot{R_0}^2(t) - 4\lambda^2 + kM^2 e^{-2R_0(t)} = 0, \tag{27}$$

$$\ddot{R_0}(t) + N^2\lambda^2\dot{y_0}^2(t) + 2\dot{R_0}^2(t) - 8\lambda^2 + kM^2 e^{-2R_0(t)} = 0. \tag{28}$$

Note that all of these equations (26), (27), and (28) are not independent. Solving Eqs. (26,27,28) gives the two types of metrics:

$$ds_S^2 = \cosh^2(\lambda N|y| - \beta)[-dt^2 + a^2(t)d\chi^2_{(3)}] + N^2dy^2, \tag{29}$$

$$ds_A^2 = \sinh^2(\lambda N|y| + \beta')[-dt^2 + a^2(t)d\chi^2_{(3)}] + N^2dy^2, \tag{30}$$

where $a(t) = e^{R_0(t)}/M$ is given by

$$a_{open}(t) = \frac{1}{\lambda}\sinh\lambda t, \quad a_{flat}(t) = \frac{1}{\lambda}e^{\lambda t}, \quad a_{closed}(t) = \frac{1}{\lambda}\cosh\lambda t, \tag{31}$$

for open, flat, and closed universe, respectively, and $\lambda = \sqrt{\frac{-\Lambda}{24M^3}}$, $\beta = -N\lambda y_0$, and $\beta' = N\lambda y_0 (> 0)$ are constants.

Let us first consider cosh (S) type solutions. The solutions consist of two branes which are located at $y = 0$ and at $y = 1$. If $\beta < 0$, one of the two branes has positive tension and the other has negative tension. On the other hand, if $N\lambda > \beta > 0$, the two branes have positive tension. If $\beta = 0$, there exist one compact positive brane solution

$$ds^2 = \cosh^2(N\lambda y)[-dt^2 + a^2(t)d\chi^2_{(3)}] + N^2dy^2, \tag{32}$$

in addition to the two brane solution, where one of the two branes has zero tension. The brane in this solution (32) is located at the boundary $y = 1$ and this surface is identified with that at $y = -1$. Due to the symmetry of the warp factor this solution satisfies S_1/Z_2 boundary condition automatically.

We now consider the boundary condition:

$$R''|_{\text{at Brane}} = -\sum_i \frac{N^2\Lambda_i}{12M^3}\delta(y - y_i), \tag{33}$$

which implies the discontinuity of R' at $y = y_i$. In Eq. (33) we have neglected terms related to R' and σ' which are finite at $y = y_i$. The resulting brane tensions satisfying this boundary condition are

$$\lambda_0 = \frac{\Lambda_0}{24M^3} = \lambda \tanh \beta, \quad \lambda_1 = \frac{\Lambda_1}{24M^3} = \lambda \tanh\left(N\lambda - \beta\right). \tag{34}$$

In the case of one brane solution, only λ_1 is meaningful. Note that all the cosmological constants λ_0 and λ_1 are smaller than λ which is in contrast to the (A) type solution [See below].

The Hubble parameters, $H = \dfrac{1}{ae^{-\sigma}}\dfrac{d(ae^{-\sigma})}{d\tau}$, where $\tau = e^{-\sigma}t$ is the proper time, at each boundaries are

$$H(0,t) = \sqrt{\lambda^2 - \lambda_0^2}\,(\coth \lambda t, 1, \tanh \lambda t), \quad H(1,t) = \sqrt{\lambda^2 - \lambda_1^2}\,(\coth \lambda t, 1, \tanh \lambda t), \tag{35}$$

for the open, flat, and closed universes, respectively. The Hubble parameters of the open and closed universe are dependent on time, but asymptotically approach to the flat universe value and the static limit ($H \to 0$) corresponds to $\lambda_i \to \lambda$ [16]. The current observation restricts the Hubble parameter of the visible brane:

$$H(0) = \sqrt{\lambda^2 - \lambda_0^2} < 10^{-60} M_P, \tag{36}$$

where $\lambda = O(M_P)$ is assumed. Although Eq. (36) appears to require highly precise fine tuning, it actually is not the case: Since the cosmological constant is related to the size of extra-dimension, this constraint only implies $\beta \simeq 60$.

Gauge hierarchy demands that the difference of the scale factors between the hidden and visible branes is of the order of 10^{16}, which means $\lambda N - 2\beta \simeq 37$. This implies the size of the extra dimension,

$$N = \frac{1}{2\lambda} \ln\left(\frac{\lambda + \lambda_0}{\lambda - \lambda_0}\,\frac{\lambda + \lambda_1}{\lambda - \lambda_1}\right) \simeq 157\ l_p. \tag{37}$$

This size of extra-dimension is a reasonable one which also guarantees the efficiency of semi-classical treatment of the universe.

Now, let us consider the sinh (A) type solutions, which are already discussed in Ref. [16] for flat spatial geometry. They admit two brane solutions with one positive and one negative tension as usual. The Hubble parameters at each boundaries are

$$H(0,t) = \sqrt{\lambda_0^2 - \lambda^2}\,(\coth \lambda t, 1, \tanh \lambda t), \quad H(1,t) = \sqrt{\lambda_0^2 - \lambda^2}\,(\coth \lambda t, 1, \tanh \lambda t), \tag{38}$$

for open, flat, and closed universe, respectively. In this case, the size of the extra-dimension to solve the gauge hierarchy problem,

$$N = \frac{1}{2\lambda} \ln\left(\frac{-\lambda_0 - \lambda}{\lambda_1 - \lambda}\,\frac{\lambda_1 + \lambda}{-\lambda_0 + \lambda}\right), \tag{39}$$

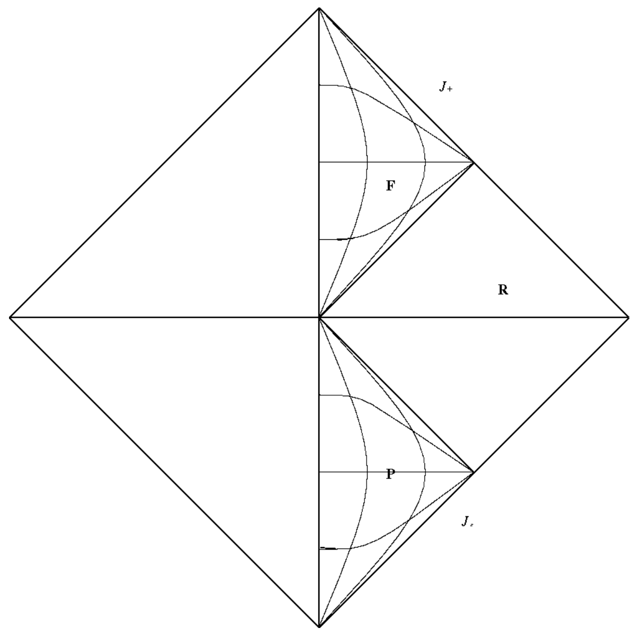

Fig. 1. Penrose diagram of the extended space.
Curves passing the origin represent $y=$ constant. The brane resides on this trajectory. $\bar{x} = 0$ line represents infinity. Inflating solution (30) is confined region (P). The metric confined in (F) is a deflating solution which can be obtained by $t \to -t$ from Eq. (30). So an inflating solution has a black hole like region, and a deflating solution has a white hole like region. The horizons $\bar{u} = 0$ or $\bar{v} = 0$ correspond to $y = 0$.

does not have a unique value, but varies depending on how λ_0 and λ_1 approach λ.

An interesting point of this solution, which has not been well discussed yet, is the existence of a horizon at $y = 0$ for $\beta = 0$ case of the bulk solution (30). For convenience, we set $N = 1$. This solution should be extendible in y coordinate. We consider only the $y - t$ space to show the extendibility and we consider only the flat space case for simplicity.

The metric of the fully extended space [See Fig. (1) for the Penrose diagram.] is given by

$$ds^2 = \frac{1}{\lambda^2}\frac{1}{\bar{x}^2}\left[-d\bar{u}d\bar{v} + d\chi^2_{(3)}\right], \tag{40}$$

which is nothing but the Randall-Sundrum solution, as can be expected from the fact that both the spaces (A) and the R-S solution have the constant negative curvature. The coordinate transformation between (40) and solution (A) is given by,

$$e^{\mp 2\lambda t} = \bar{u}\bar{v}, \quad \cosh(N\lambda y) = \mp\frac{\bar{t}}{\bar{x}}, \tag{41}$$

where the negative sign corresponds to the inflating universe (P) and the positive sign corresponds to the deflating universe (F). The deflating universe solution can be obtained

by setting $t \to -t$ in Eq. (30). These inflating and deflating solutions cover only the region $\bar{u},\ \bar{v} < 0$ or $\bar{u},\ \bar{v} > 0$ of the whole space, respectively. The inverse transformations are

$$\bar{x} = \mathrm{cosech}(N\lambda y)\ e^{-\lambda t}, \qquad \bar{t} = \coth(N\lambda y)\ e^{-\lambda t}. \tag{42}$$

The metric of a localized observer in region (R) is given by

$$ds^2 = N^2 \sin^2 \lambda\tau\ dy^2 + e^{-2N\lambda|y|} \left[-e^{-2N\lambda|y|}\ d\tau^2 + \frac{1}{\lambda^2} \sin^2 \lambda\tau\ d\chi^2_{(3)} \right]. \tag{43}$$

The coordinate transformation between Eqs. (40) and (43) is given by

$$-\bar{u}\bar{v} = e^{-2\lambda N y}, \quad \frac{\bar{t}}{\bar{x}} = -\cos \lambda\tau, \tag{44}$$

where the ranges of t and y are $0 \leq \tau \leq \pi/\lambda$ and $-\infty < y < \infty$, respectively. The scale factor of the visible brane expands to $e^{-2N\lambda}/\lambda$ and then recontract to zero within a finite time. The Hubble parameter does not depends on y coordinates and is

$$H(t) = \lambda \cot \lambda\tau. \tag{45}$$

If some brane universe lives in this space, its 5-th dimensional size expands in time and then recontract. So this configuration does not have stable brane universe solution.

This diagram [Fig.(1)] opens us another interesting possibility: R-S brane as a wormhole. Consider two inflating brane worlds at (P) and (F). These two branes with small positive tensions will change background geometry somewhat, but we assume that this change is small enough not to destroy its causal structure. Then assume a third brane which is the R-S brane. R-S brane passes both regions (P) and (F) and meets at some points with the two brane worlds at (P) and (F). If one can move from one brane to another brane when the two branes cross, the R-S brane can be used as a wormhole between the two branes, which do not cross each other.

Some general remarks are in order. First, the limit, $\lambda \to 0$ and large $\lambda N y$, of the open universe solution for (S) is

$$ds^2_{(S)} = e^{-2s|y|} \left[-dt^2 + t^2 d\chi^2_{(3)} \right] + N^2 dy^2. \tag{46}$$

The corresponding solution for the flat universe is the R-S one and there is no solution for the closed universe. The cosmological constant of each branes are given by Eq. (2).

The second is that 05 part of the Einstein equation, which comes from the variation of N_a in action (9),

$$R' \frac{N'}{N} - \dot{R}\sigma' - \dot{R}R' - \dot{R}' = 0, \tag{47}$$

is automatically satisfied with our solutions. This equation can be used to obtain more general solutions than Eqs. (30) and (29): For example, by imposing only the stabilization condition $N' = 0$, one gets

$$\sigma(t,y) + R(t,y) = -\ln \dot{R}(t,y) + f(t), \tag{48}$$

or $R' = -\sigma'$, instead, gives

$$N(t,y) = R'(t,y)g(y), \tag{49}$$

which may allow more general solutions.

4 Summary and Discussion

We have constructed the brane-world solutions of the Robertson-Walker type by writing the effective action of a brane world in $4+1$ splitting form. We have obtained spatially open, flat, and closed universe solutions which are inflating in t and static in y direction. There exist two types of solutions (S) and (A). Notably, the one positive-tension brane universe solution is admitted for (S) type as well as two brane solutions with two positive tension branes or one positive and one negative tension branes. We also estimate a reasonable size of the 5-th dimension which solves the cosmological constant and the M_P/M_W hierarchy problems. Finally, in the case of (A) type solution, we obtain its covering space which is nothing but the R-S solution. Moreover, this (A) type solutions have a white-hole like region or a black-hole like region.

We finally note that there is a minimal size for the closed universe solution in Eqs. (29) and (30). This size is $\sinh(\beta)/\lambda$ or $\cosh(\beta)/\lambda$ and determined by the cosmological constant of the bulk and the tension of the branes:

$$L_{\min,A} = \frac{\lambda_0}{\lambda}\frac{1}{\sqrt{\lambda^2 - \lambda_0^2}}, \qquad L_{\min,S} = \frac{1}{\sqrt{\lambda^2 - \lambda_0^2}}. \tag{50}$$

These sizes are very large $[O(10^{60}\ l_P)]$ if the condition (36) is satisfied and it seems that the closed universe is excluded as a candidate for our universe. However, it is not evident whether or not the cosmological constant satisfies Eq. (36) even at the initial epoch of the universe. Moreover, if there is matter on the bulk at the initial epoch, the contribution of matter will dominate k term. Then the possibility of the closed universe can not be excluded.

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References

[1] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **B 125**, 136 (1983).

[2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B 429** , 263 (1998), hep-ph/9803315.

[3] G. F. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys. **B 544**, 3 (1999), hep-ph/9811291.

[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999), hep-ph/9905221.

[5] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999), hep-th/9906064.

[6] P. Horava and E. Witten, Nucl. Phys. B460, 506 (1999).

[7] M. Cvetic and H. Soleng, Phys. Rep. **282**, 159 (1997).

[8] A. Kehagias, Phys. Lett. B469, 123 (1999), hep-th/9906204.

[9] H. Verlinde, *Holography and compactification*, hep-th/9906182.

[10] J. Lykken and L. Randall, hep-th/9908076.

[11] O. DeWolfe, D.Z. Freedman, S.S. Gubser, and A. Karch, hep-th/9909134.

[12] P. Kanti, I.I. Kogan, K.A. Olive, and M. Pospelov, Phys. Lett. **B 468**, 31 (1999), hep-ph/9909481; Phys. Rev. **D 61** , 106004 (2000), hep-ph/9912266; hep-ph/0005146 ; P. Kanti, K. A. Olive, and M. Pospelov, *Solving the Hierarchy Problem in Two-Brane Cosmological Models*, hep-ph/0005146.

[13] H.B. Kim, Phys. Lett. **B 478** 285 (2000), hep-th/0001209.

[14] N. Nihei, Phys. Lett. **B 465** 81 (1999), hep-ph/9905487.

[15] N. Kaloper, Phys. Rev. **D 60** 123506 (1999), hep-th/9905210.

[16] Hang Bae Kim and Hyung Do Kim, Phys. Rev. **D 61**, 064003 (2000), hep-th/9909053.

[17] N. Kaloper and A. Linde, Phys. Rev. **D 59** (1999) 101303, ; P.Binétury, C. Deffayet, and D. Langlois, hep-th/9905012.

[18] C. CsáKi, M. Graesser, C. Kolda, and J. Terning, Phys. Lett. **B 462** 34 (1999), hep-ph/9906513.

[19] J. M. Cline, C. Grojean, and G. Servant, Phys. Rev. Lett. **83**, 4245 (1999), hep-ph/9906523.